Adaptive Control

Chapter 14: Adaptive regulation – Rejection of unknown disturbances

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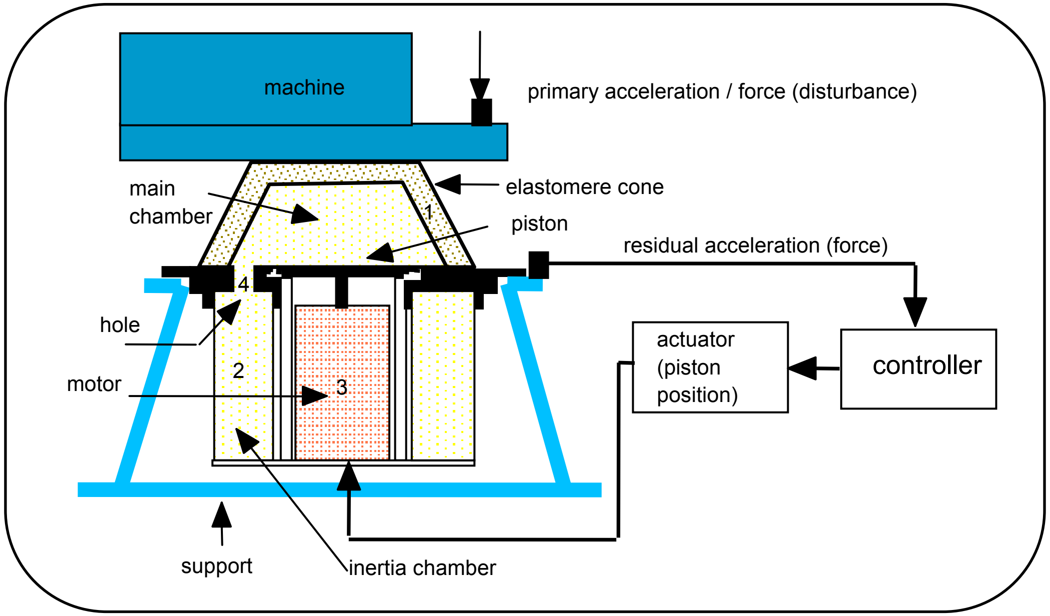
Adaptive regulation – Rejection of unknown disturbances

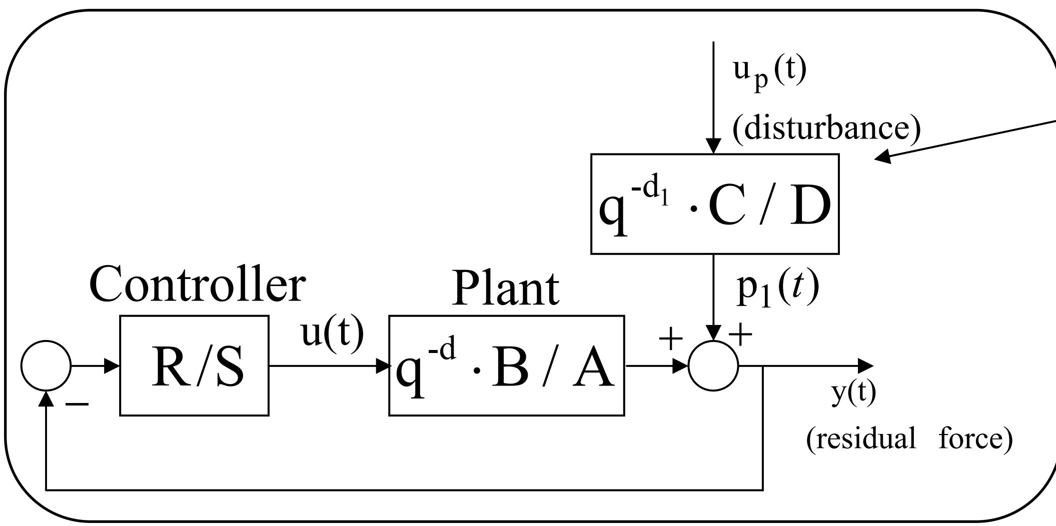
Abstract This chapter addresses the problem of attenuation (rejection) of unknown disturbances without measuring them by using a feedback approach. In this context the disturbance model is unknown and time varying while the model of the plant is known (obtained by system identification) and almost invariant. This requires an adaptive approach. The term "adaptive regulation" has been coined to characterize this control paradigm. Direct and indirect adaptive regulation strategies using the internal model principle and the Youla-Kucera parameterization will be presented. The evaluation of the methodology is done in real time on an active vibration control system using an inertial actuator.

The Active Suspension System



Objective:

- **Reject the effect of unknown and variable narrow band disturbances**
- **Do not use an aditional measurement**

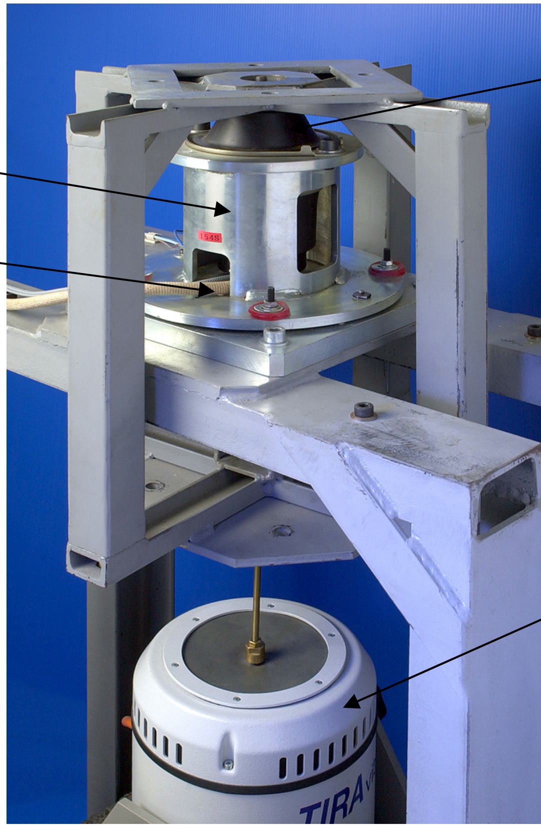


Two paths :

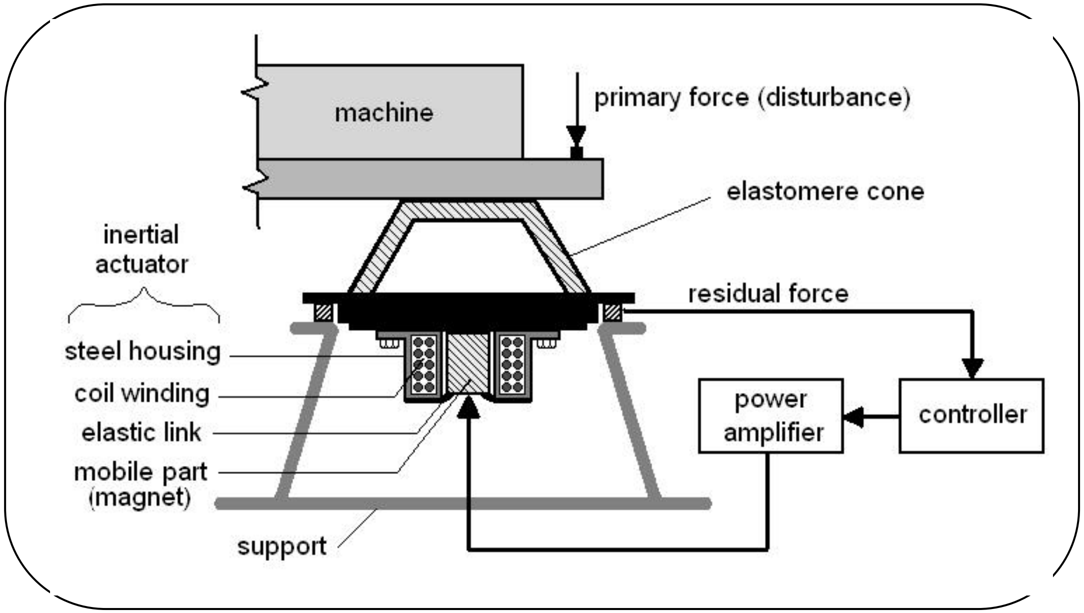
- Primary
- Secondary (double differentiator)

$$T_s = 1.25\,\text{ms}$$

The Active Suspension



The Active Vibration Control with Inertial Actuator

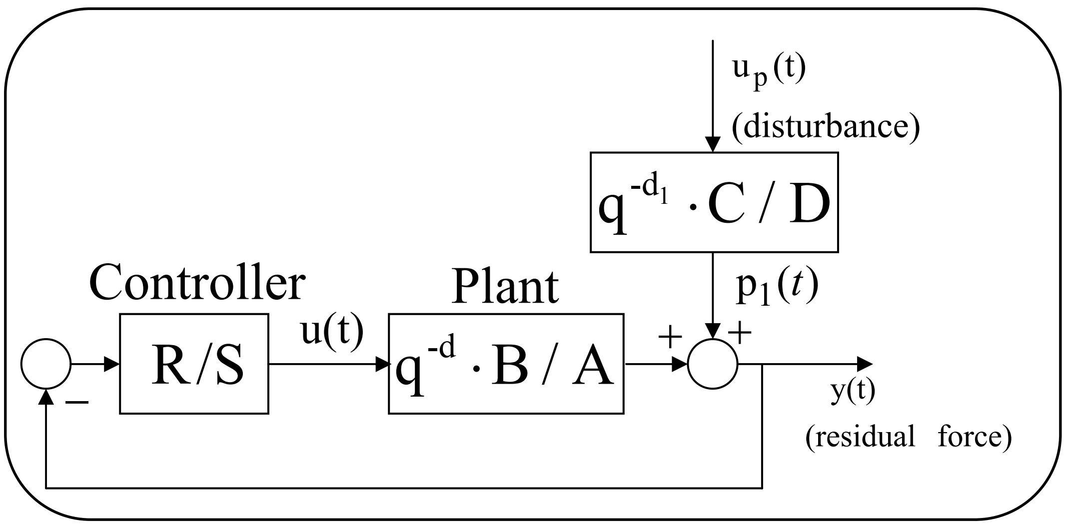


Objective:

•Reject the effect of unknown and variable narrow band disturbances

•Do not use an aditional measurement

Same control approach but different technology

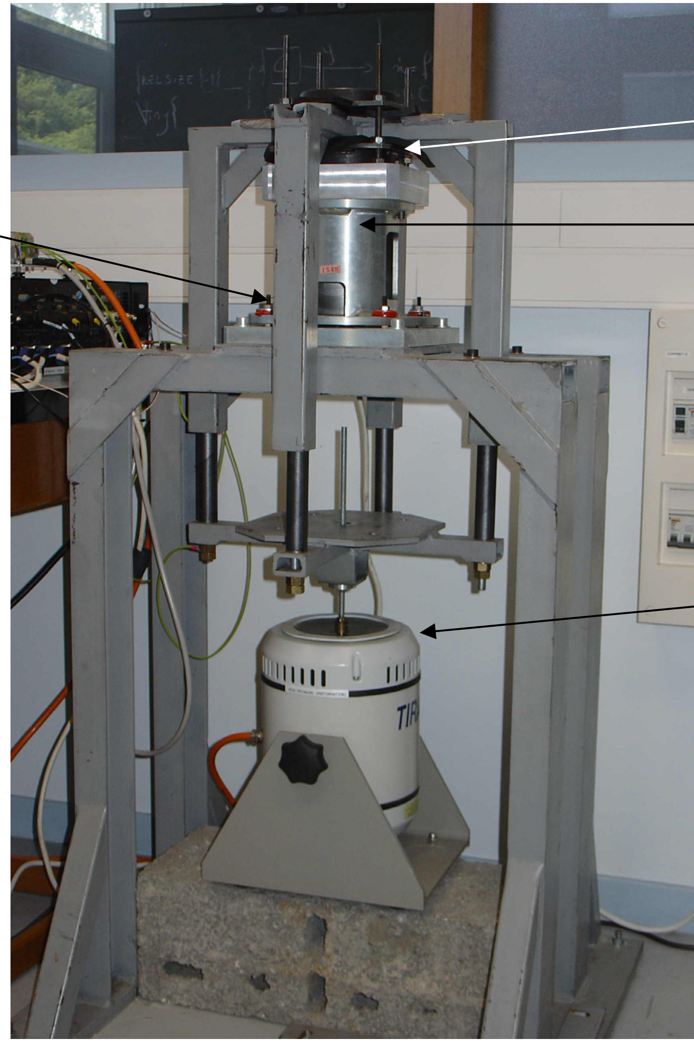


Two paths :

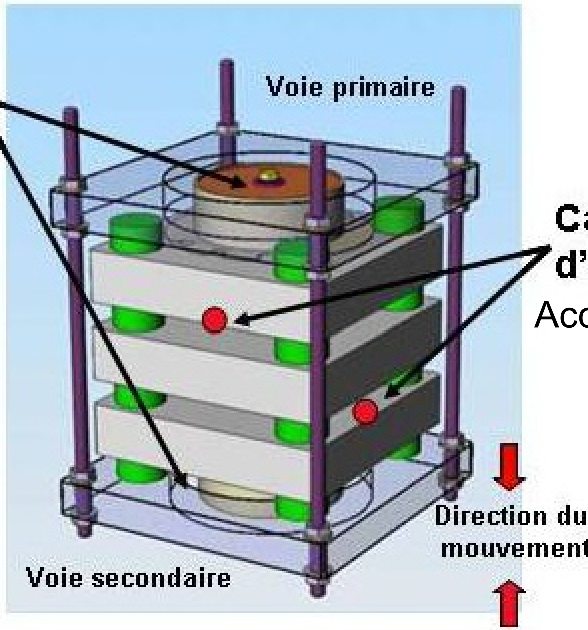
- •Primary
- •Secondary (double differentiator)

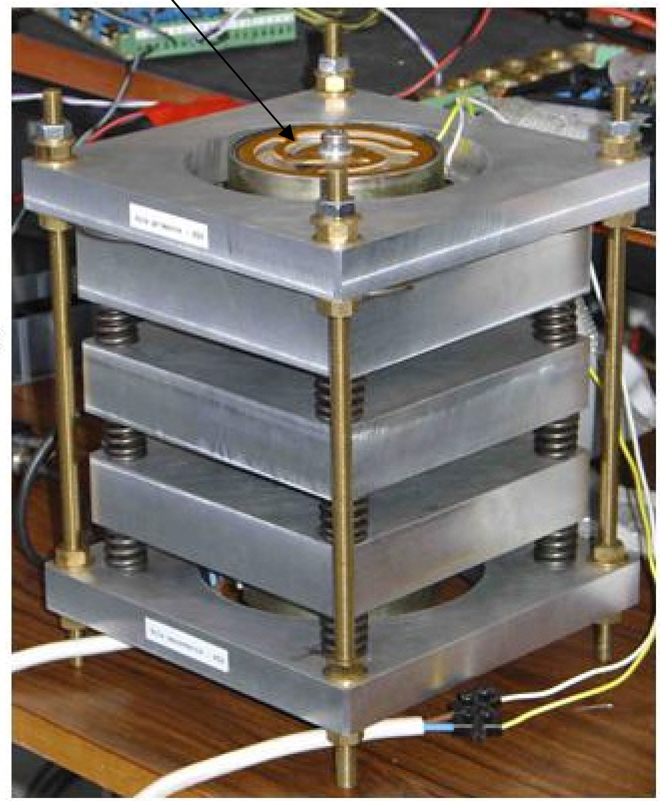
$$T_s = 1.25\ \mathrm{ms}$$

View of the active vibration control with inertial actuator



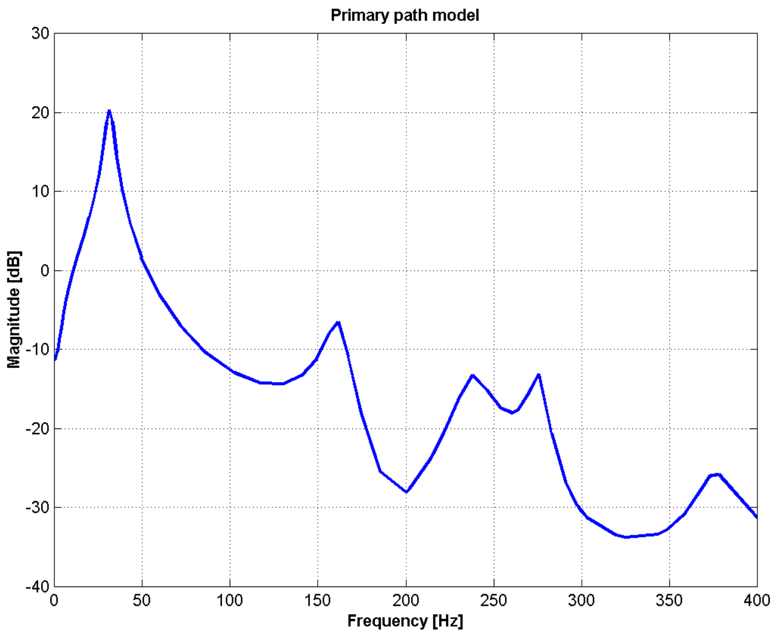
View of a flexible controlled structure using inertial actuators



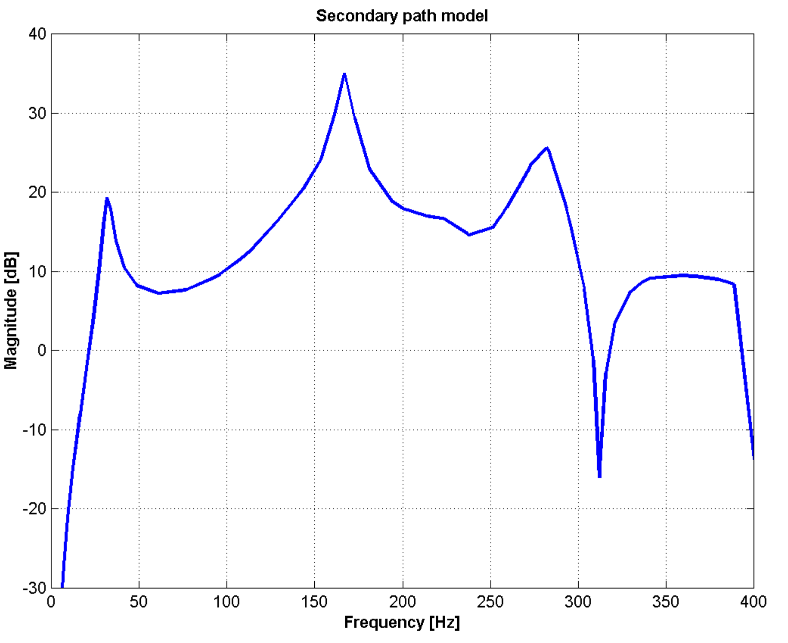


Frequency Characteristics of the Identified Models

Primary path



Secondary path

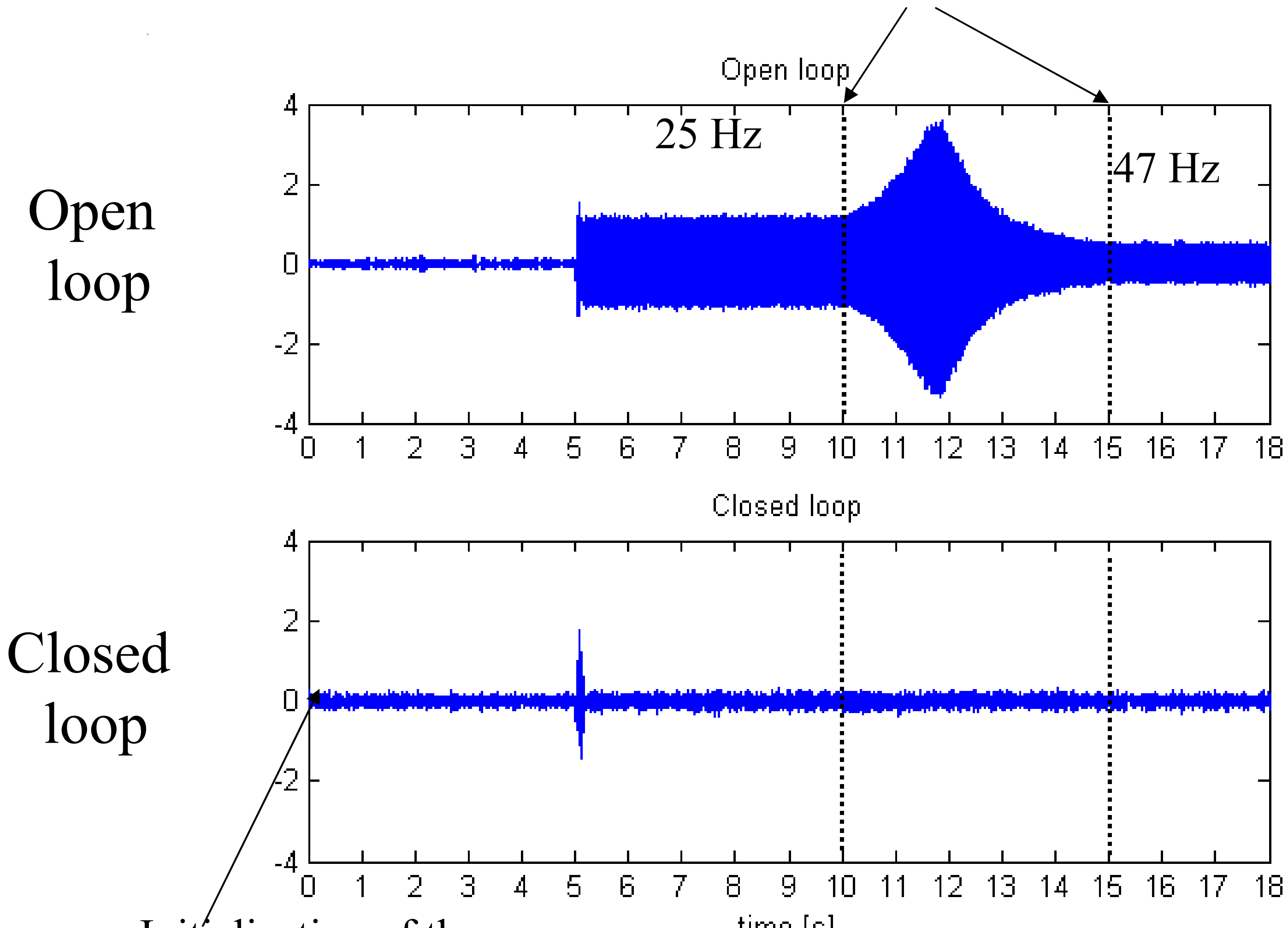


$$n_A = 14\,;\, n_B = 16\,;\, d = 0$$

Further details can be obtained from : http://iawww.epfl.ch/News/EJC_Benchmark/

Direct Adaptive Control : disturbance rejection

Disturbance : Chirp

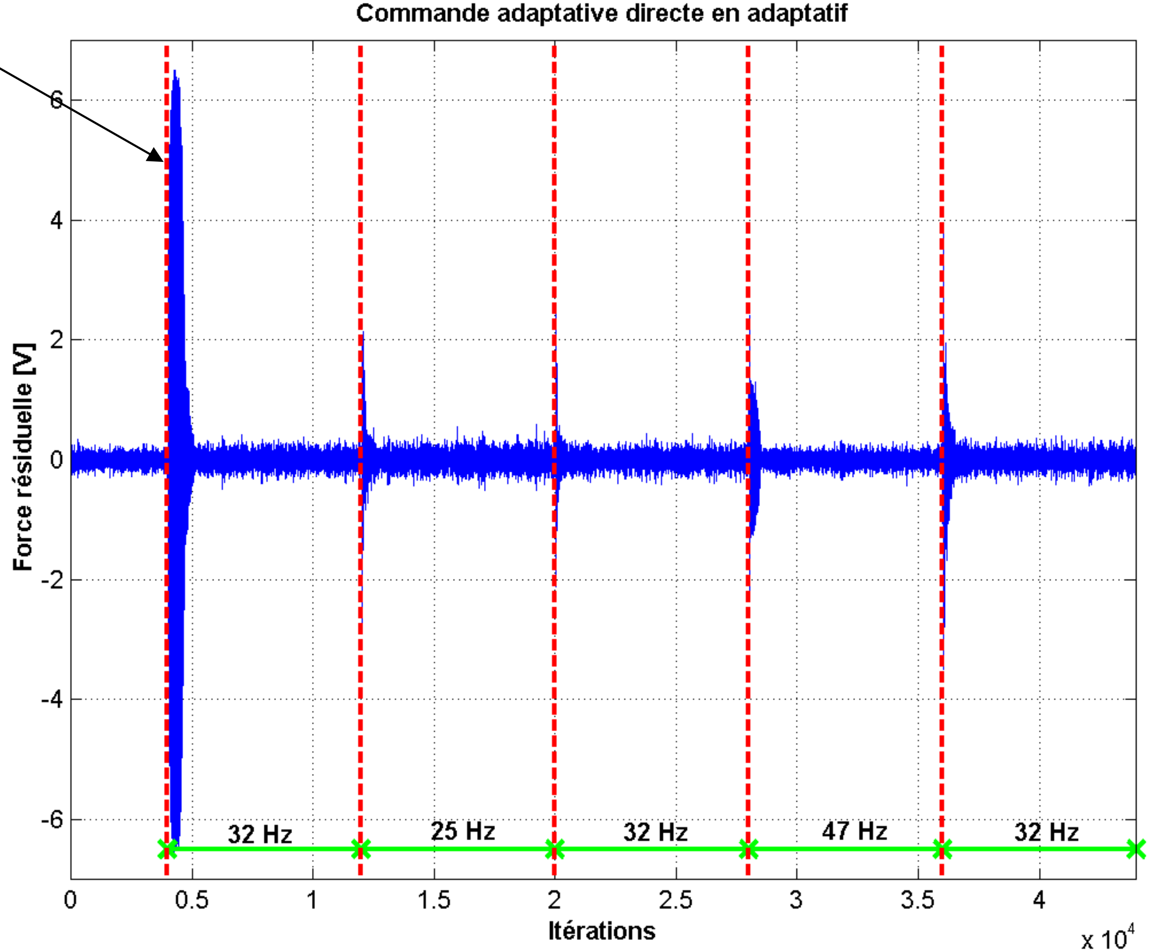


Initialization of the adaptive controller

Time Domain Results
Adaptive Operation

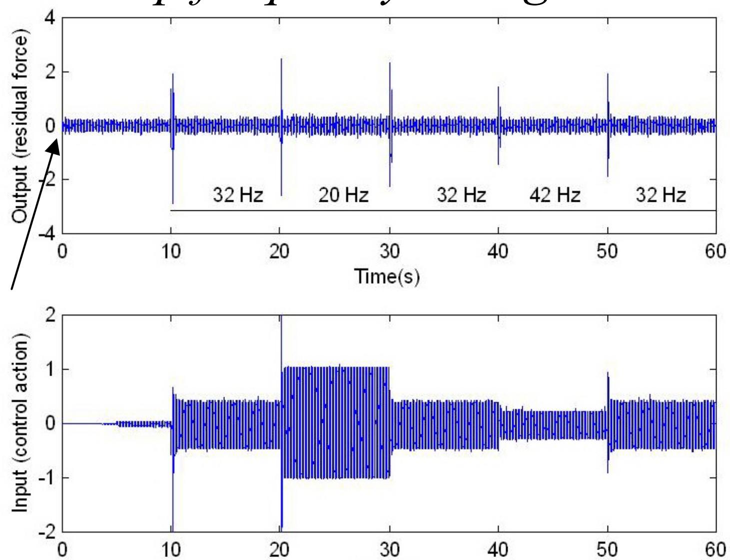
Simultaneous controller initialization and disturbance application

Direct adaptive control



Direct Adaptive Control

Step frequency changes



Initialization of the adaptive controller

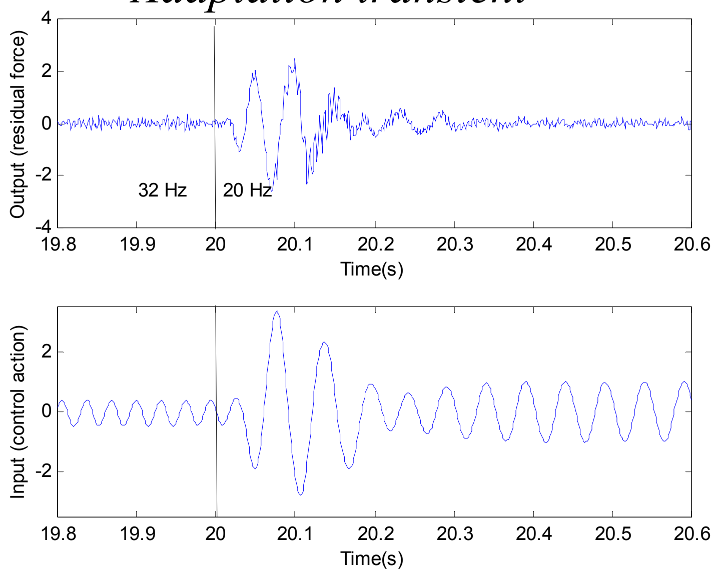
Output

Input

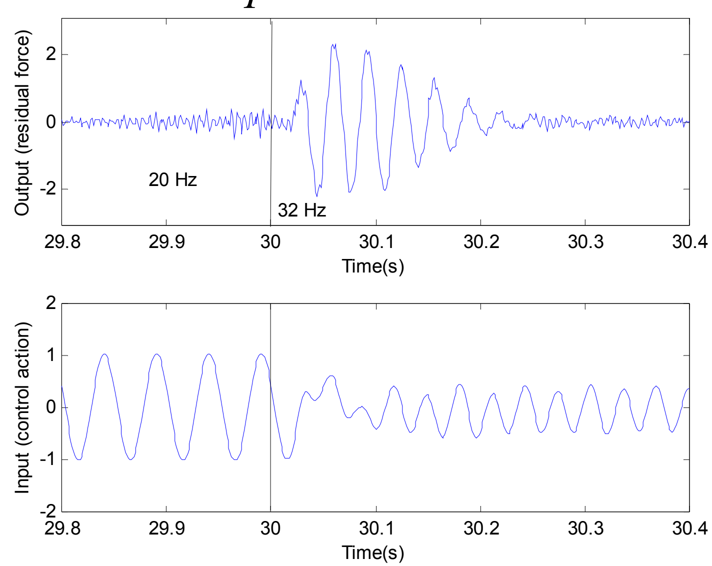
Adaptation transient

output

input



Adaptation transient



Rejection of unknown finite band disturbances

- **Assumption:** Plant model almost constant and known (obtained by system identification)
- **Problem:** Attenuation of unknown and/or variable stationary disturbances without using an additional measurement
- **Solution:** Adaptive feedback control
  - Estimate the model of the disturbance (indirect adaptive control)
  - Use the internal model principle
  - Use of the Youla parameterization (direct adaptive control)

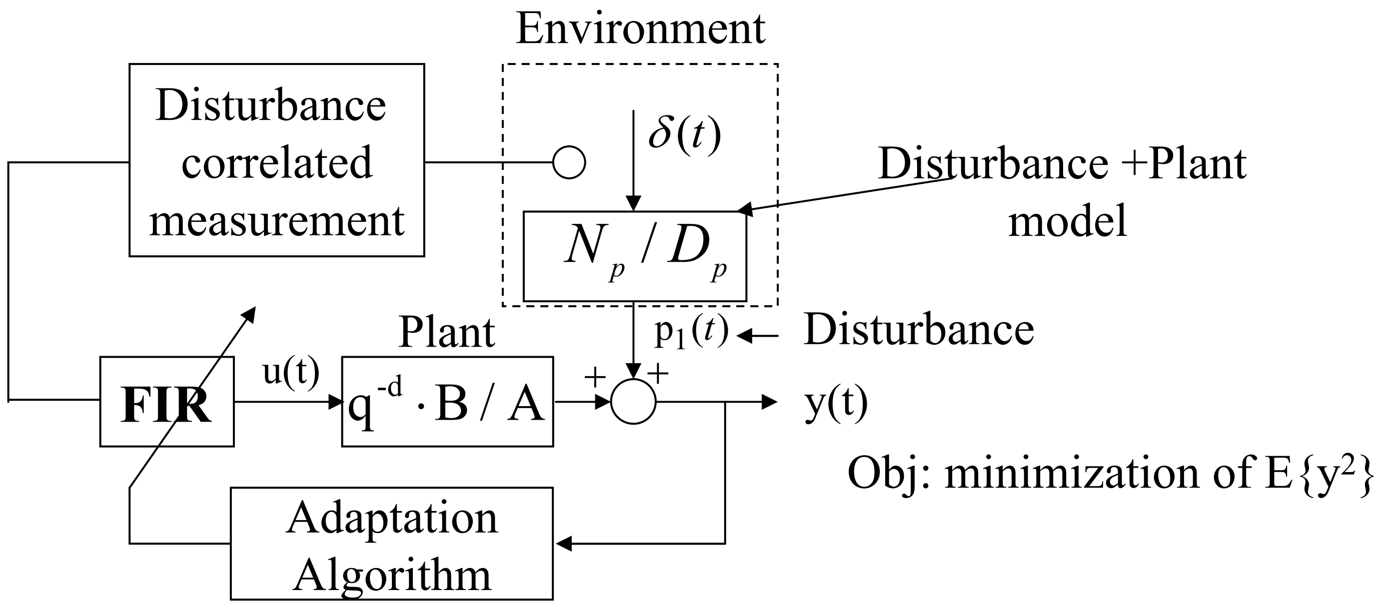
A robust control design should be considered assuming that the model of the disturbance is known

***A class of applications**: suppression of unknown vibration (active vibration control)*

***Attention:** The area was "dominated " by adaptive signal processing solutions (Widrow's adaptive noise cancellation) which require an additional transducer*

***Remainder :** Models of stationary sinusoidal disturbances have poles on the unit circle*

Unknown disturbance rejection – classical solution

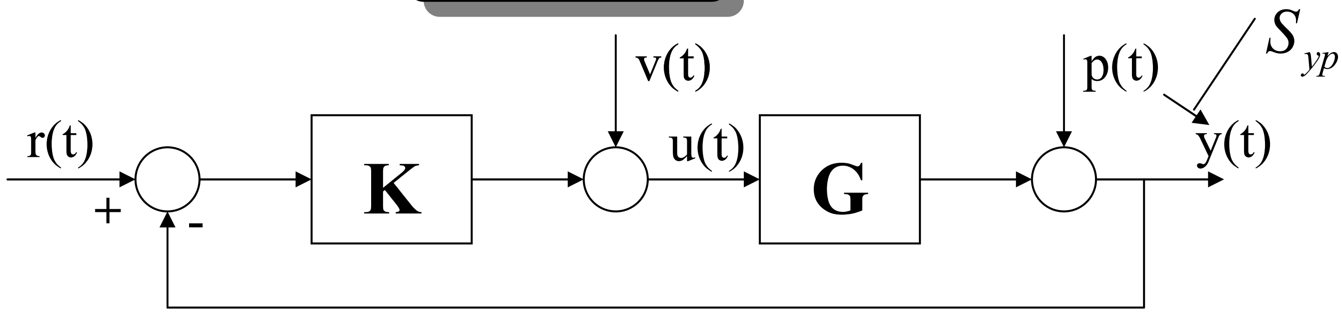


Disadvantages:

- requires the use of an additional transducer
- difficult choice of the location of the transducer
- adaptation of many parameters

Not justified for the rejection of narrow band disturbances

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = \frac{R'(q^{-1})H_R(q^{-1})}{S'(q^{-1})H_S(q^{-1})}$$

H_R and H_S are pre-specified

Output Sensitivity function :

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S'(z^{-1})H_S(z^{-1})}{P(z^{-1})}$$

Closed loop poles : $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

The gain of S_{yp} is zero at the frequencies where $S_{yp}(e^{-j\omega})=0$ (perfect attenuation of a disturbance at this frequency)

Disturbance model

Deterministic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance}$$

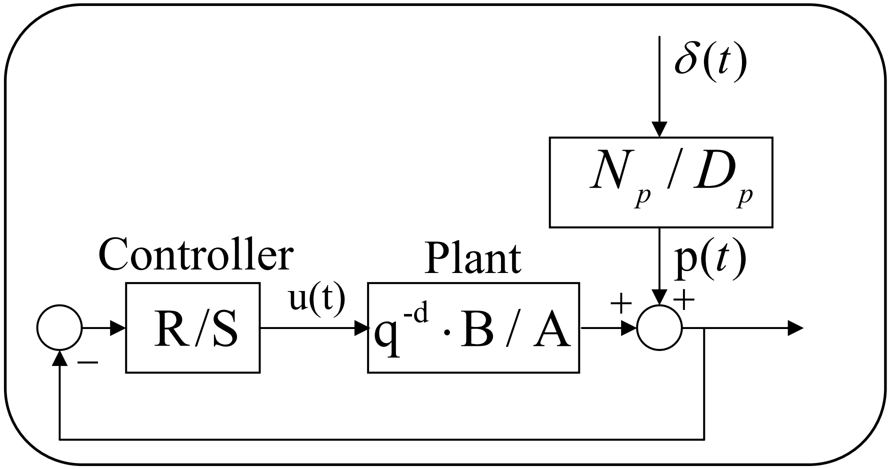
$D_p \rightarrow$ poles on the unit circle; $\delta(t)$ $=$ Dirac

Stochastic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot e(t) : \text{stochastic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $e(t)$ $=$ Gaussian white noise sequence $(0, \sigma)$

Closed loop system. Notations



$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $\delta(t) = $ Dirac

Controller :

$$R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$$

$$S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$$

Internal model principle: $H_S(z^{-1}) = D_p(z^{-1})$

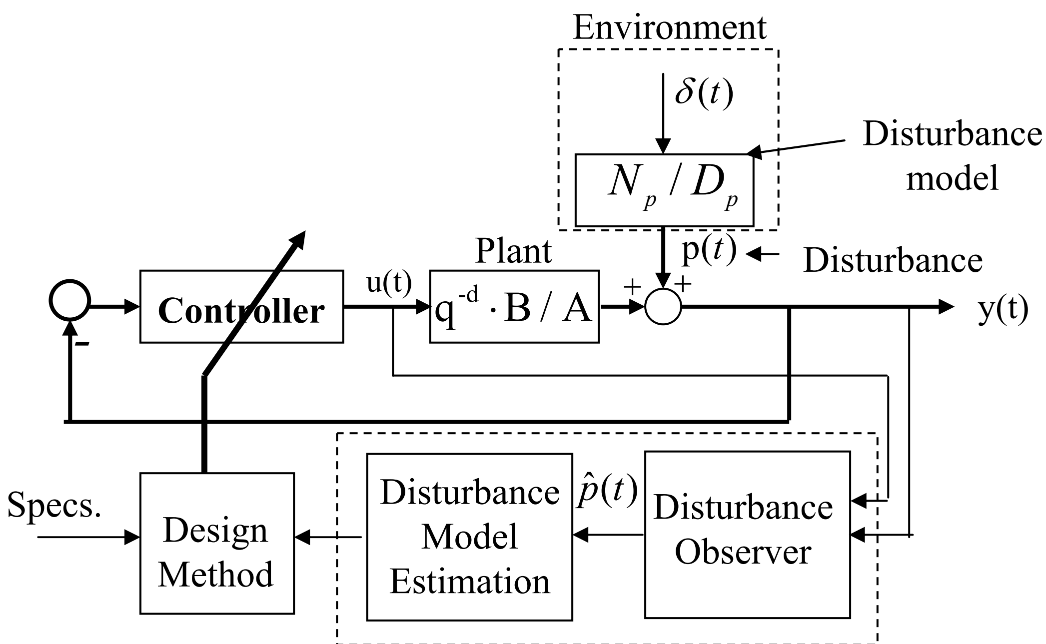
Output : $y(t) = \dfrac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) \longrightarrow y(t) = \dfrac{A(q^{-1})H_S(q^{-1})S'(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \dfrac{1}{D_p(q^{-1})} \cdot \delta(t)$

CL poles : $P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d}B(q^{-1})R(q^{-1})$

Indirect adaptive regulation

Two-steps methodology:

1. Estimation of the disturbance model, $D_p(q^{-1})$
2. Computation of the controller, imposing $H_S(q^{-1}) = \hat{D}_p(q^{-1})$



It can be time consuming (if the plant model B/A is of large dimension)

Indirect adaptive control

Step I : Estimation of the disturbance model

ARMA identification (Recursive Extended Least Squares)

Step II: Computation of the controller

Solving Bezout equation (for S' and R)

$$H_S = \hat{D}_p$$

$$A\hat{D}_p S' + q^{-d} BR = P$$

$$S = \hat{D}_p S'$$

Remark:

It is time consuming for large dimension of the plant model

Internal model principle and Q-parametrization)

Central contr: $[R_0(q^{-1}),S_0(q^{-1})]$.

CL poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\, u(t) = -R_0(q^{-1})\, y(t)$

Q-parametrization :

$\boldsymbol{R(z^{1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1});}$

$\boldsymbol{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1}).}$

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + ...q_{n_Q} q^{-n_Q}$$

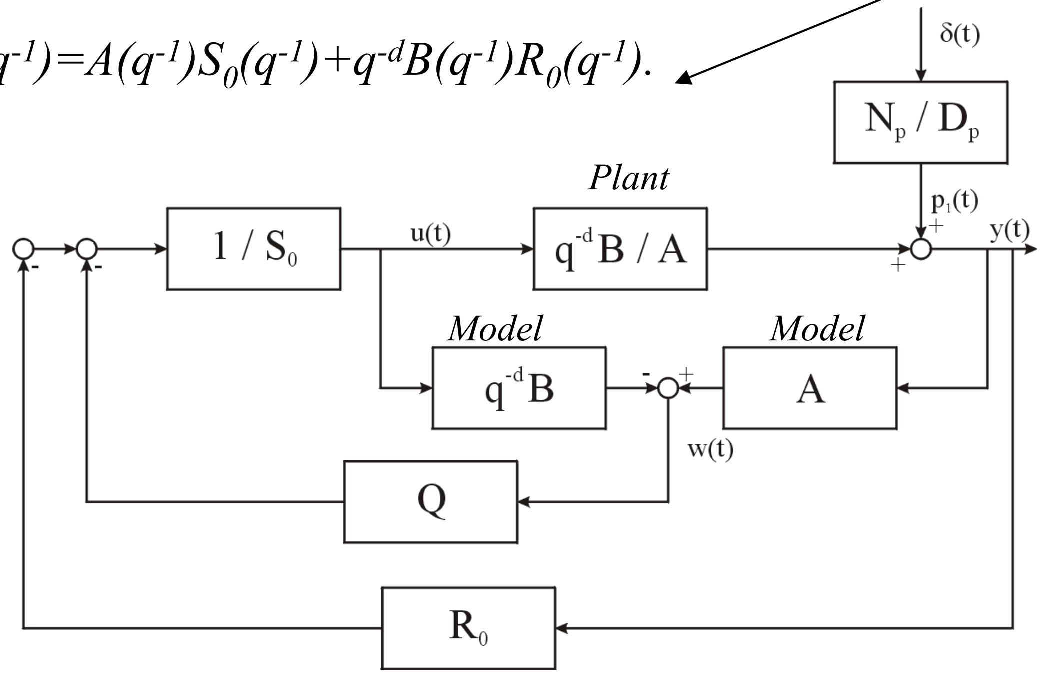
Control: $S(q^{-1})u(t) = -R(q^{-1})y(t)$

$$S_0(q^{-1})\, u(t) = -R_0(q^{-1})\, y(t) - Q(q^{-1})\, w(t),$$

where $w(t) = A(q^{-1})\, y(t) - q^{-d}B(q^{-1})\, u(t)$.

CL poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

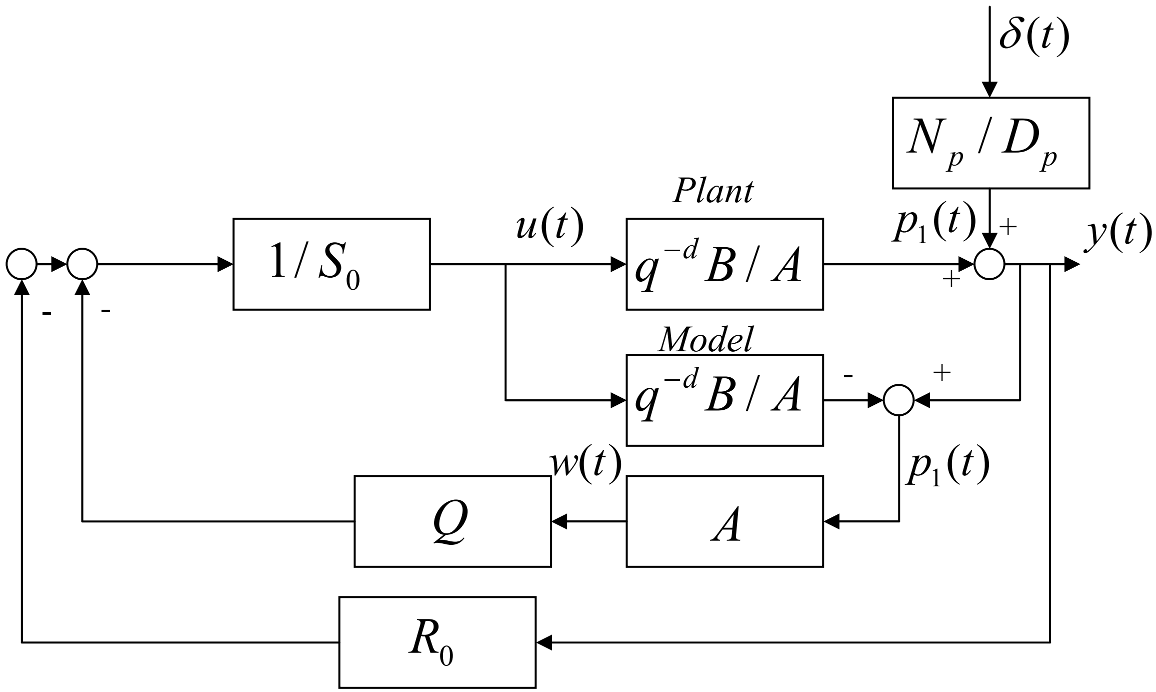
The closed loop poles remain unchanged



Model = Plant

$w=Ap_1$

Yula-Kucera parametrization
An interpretation for the case *A* asympt. stable



$$w(t) = \frac{AN_p}{D_p}\delta(t)$$

Internal model principle and Q- parameterization

Central contr: $[R_0(q^{-1}),S_0(q^{-1})]$.

CL Poles: $P(q^{-1})=A(q^{-1})S_0(q^{-1})+q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1})\, u(t) = -R_0(q^{-1})\, y(t)$

Q-parameterization :

$\mathbf{R(z^{-1})=R_0(q^{-1})+A(q^{-1})Q(q^{-1})}$;

$\mathbf{S(q^{-1})=S_0(z^{-1})-q^{-d}B(q^{-1})Q(q^{-1})}$.

Closed Loop Poles remain unchanged

Internal model assignment on Q *(find Q such that S contains the disturbance model):*

$$S = S_0 - q^{-d} BQ = MD_p \quad \Rightarrow \quad \textit{Solve:} \quad \underset{?}{M}D_p + q^{-d} B\underset{?}{Q} = S_0$$

Will lead also to an « indirect adaptive control solution »

BUT:

*Q can be used to "**directly**" tune the internal model without changing the closed loop poles(see next)*

Direct Adaptive Control (unknown D_p)

Hypothesis: Identified (known) plant model *(A,B,d)*.

Goal: minimize *y(t)* (according to a certain criterion).

Consider $p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t)$: deterministic disturbance. $w(t) = \frac{AN_p}{D_p}\delta(t)$

$S(q^{-1})$

$$y(t) = \frac{A(q^{-1})\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = \frac{\left[S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1})\right]}{P(q^{-1})} w(t)$$

$$w(t) = \frac{A(q^{-1})N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = A(q^{-1}) \cdot y(t) - q^{-d} \cdot B(q^{-1}) \cdot u(t)$$

Define (construct): $$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t).$$

Define $\varepsilon^0(t+1)$ as the value of $y(t+1)$ obtained with $\hat{Q}(t,q^{-1})$

$$\varepsilon^0(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\hat{Q}(t,q^{-1}) \cdot w(t)$$

One can define now the *a posteriori* error (using $\hat{Q}(t+1,q^{-1})$) as:

$$\varepsilon(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1) - \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\hat{Q}(t+1,q^{-1}) \cdot w(t) \quad (*)$$

We need to express ε(t)as:

$$\varepsilon(t+1) = \left[Q(q^{-1}) - \hat{Q}(t+1,q^{-1})\right]\Psi(t)$$

Using: $MD_p + q^{-d}BQ = S_0$, (*) becomes

Vanishing term

Leads to a direct adaptive control

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1,q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t+1)$$

Details: $$v(t+1) = \frac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})}w(t+1) = \frac{M(q^{-1})N_p(q^{-1})}{P(q^{-1})}\delta(t+1)$$

$$\frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{q^{-d}B(q^{-1})}{P(q^{-1})} \cdot w(t+1)$$

Instead of solving $MD_p + q^{-d}BQ = S_0$ *search recursively for :*

$$\hat{Q}(t,q^{-1})^* = \arg\min_{\hat{Q}} \sum_{i=0}^{t} \varepsilon^2\left[i,\hat{Q}\right]$$

The Algorithm

$$w(t+1) = A(q^{-1})\underbrace{y(t+1)}_{measured} - q^{-d}B^*(q^{-1})\underbrace{u(t)}_{measured};\ \ (B(q^{-1})u(t+1) = B^*(q^{-1})u(t))$$

define :

$$\varepsilon^\circ(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}w(t+1) - \hat{Q}(t,q^{-1})\frac{q^{-d}B(q^{-1})}{P(q^{-1})}w(t+1).$$

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})}\cdot w(t+1); \quad w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})}\cdot w(t);$$

$$\hat{\theta}^T(\mathrm{t}) = \left[\hat{\mathrm{q}}_0(t) \quad \hat{\mathrm{q}}_1(t)\right]; \quad \phi^T(t) = \left[w_2(t) \quad w_2(t-1)\right], \quad (\text{for} \quad n_{D_P} = 2 \ \text{since}\ n_Q = n_{D_P}\text{-1})$$

A priori adaptation error : $\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t)$

A posteriori adaptation error : $\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t)$

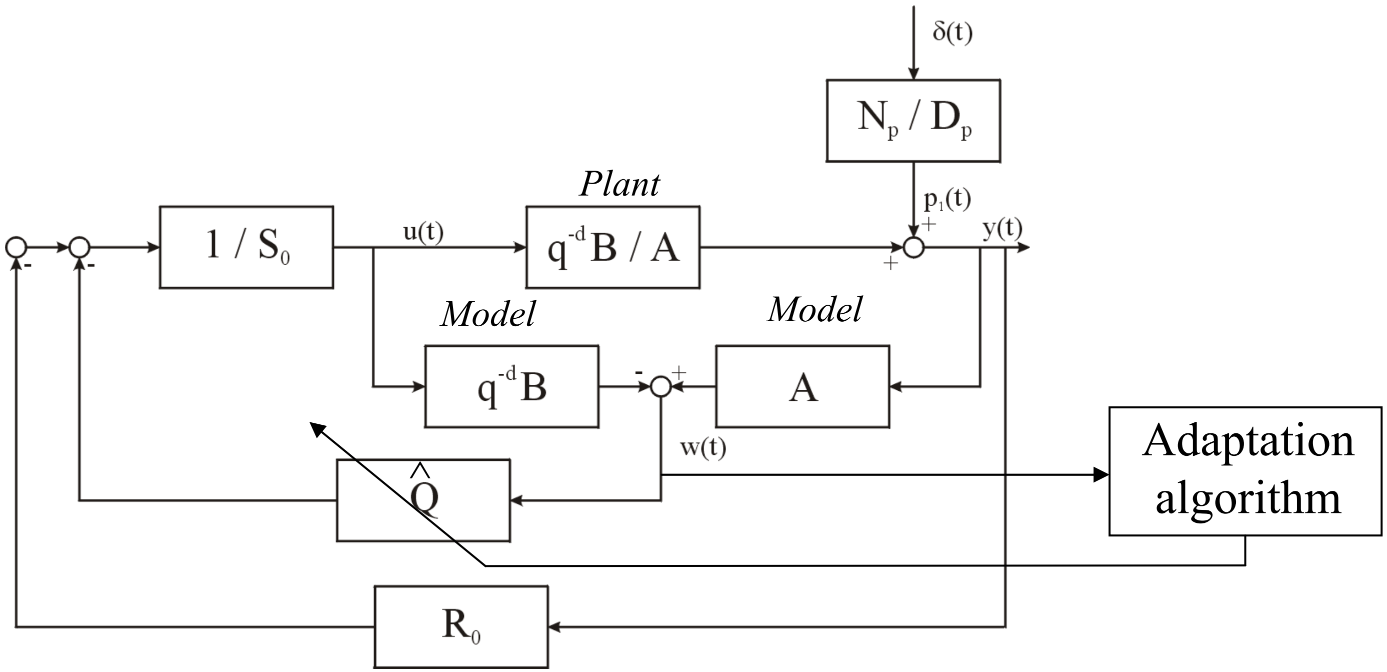
Parameter adaptation algorithm:

$$\begin{cases} \hat{\theta}(\mathrm{t}+1) = \hat{\theta}(\mathrm{t}) + \mathrm{F}(\mathrm{t}+1)\phi(\mathrm{t})\varepsilon^0(\mathrm{t}+1); \\ F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t). \end{cases}$$

Various choices possible for λ_1 and λ_2 *which define the adaptation gain time profile*

(For a stability proof see Automatica, 2005, no.4 pp. 563-574)

Direct adaptive rejection of unknown disturbances



• *The order of the Q polynomial depends upon the order of the disturbance model denominator (D_P) and not upon the complexity of the plant model*
• *Less parameters to estimate then for the identification of the disturbance model*
• *Operation in "self –tuning" mode (constant unknown disturbance) or "adaptive" mode (time varying unknown disturbance)*

Further experimental results on the active suspension
Comparison between direct/indirect adaptive control

Real-time results – Active Suspension (continuation)

Narrow band disturbances = variable frequency sinusoid $\Rightarrow n_Q = 1$

Frequency range: 25 ÷ 47 Hz

Evaluation of the two algorithms in **real-time**

Nominal controller $[R_0(q^{-1}), S_0(q^{-1})]$: $n_{R0}=14$, $n_{S0}=16$

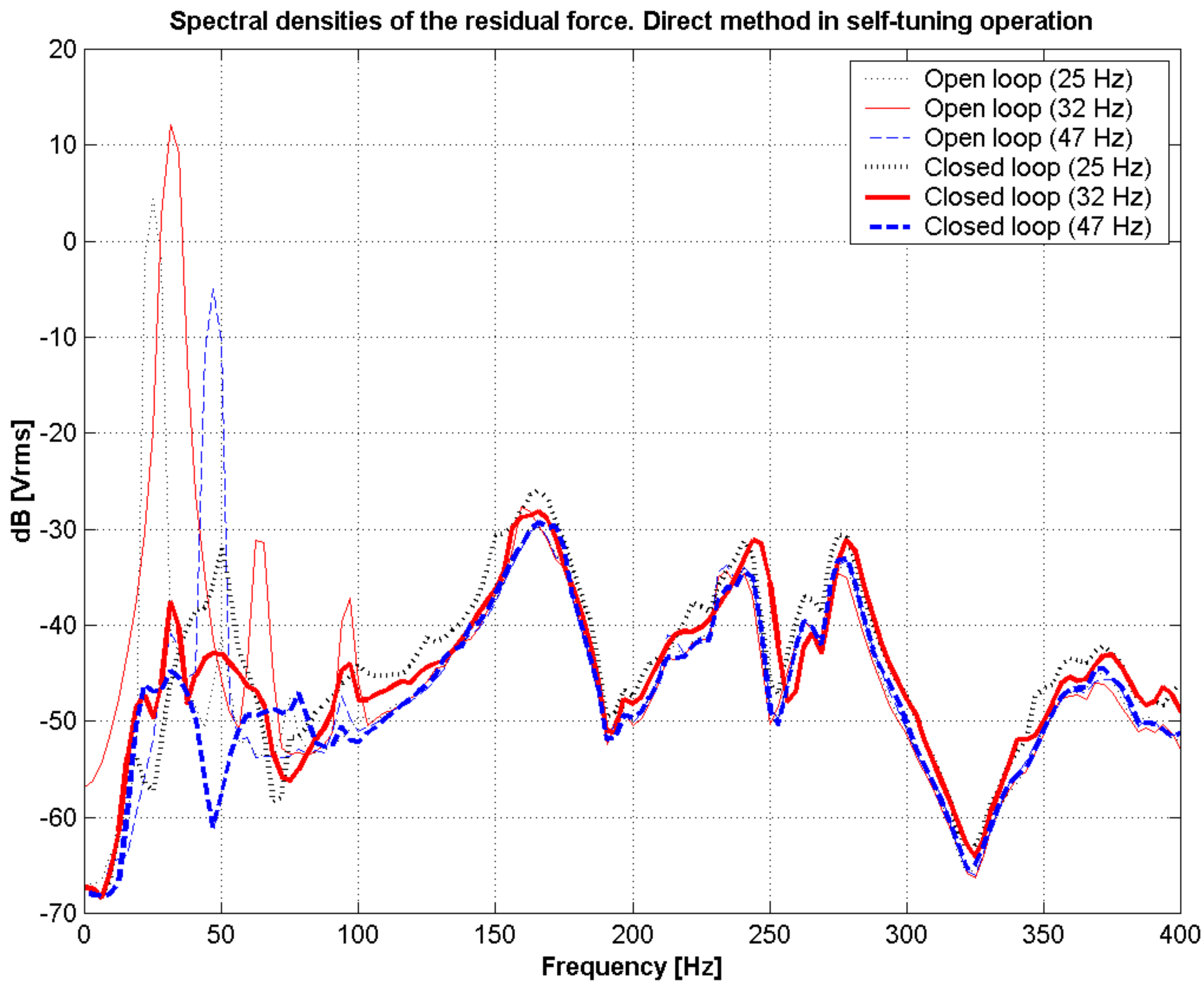
Implementation protocol 1: Self-tuning

- The algorithm stops when it converges and the controller is applied.
- It restarts when the variance of the residual force is bigger than a given threshold.
- As long as the variance is not bigger than the threshold, the controller is constant.

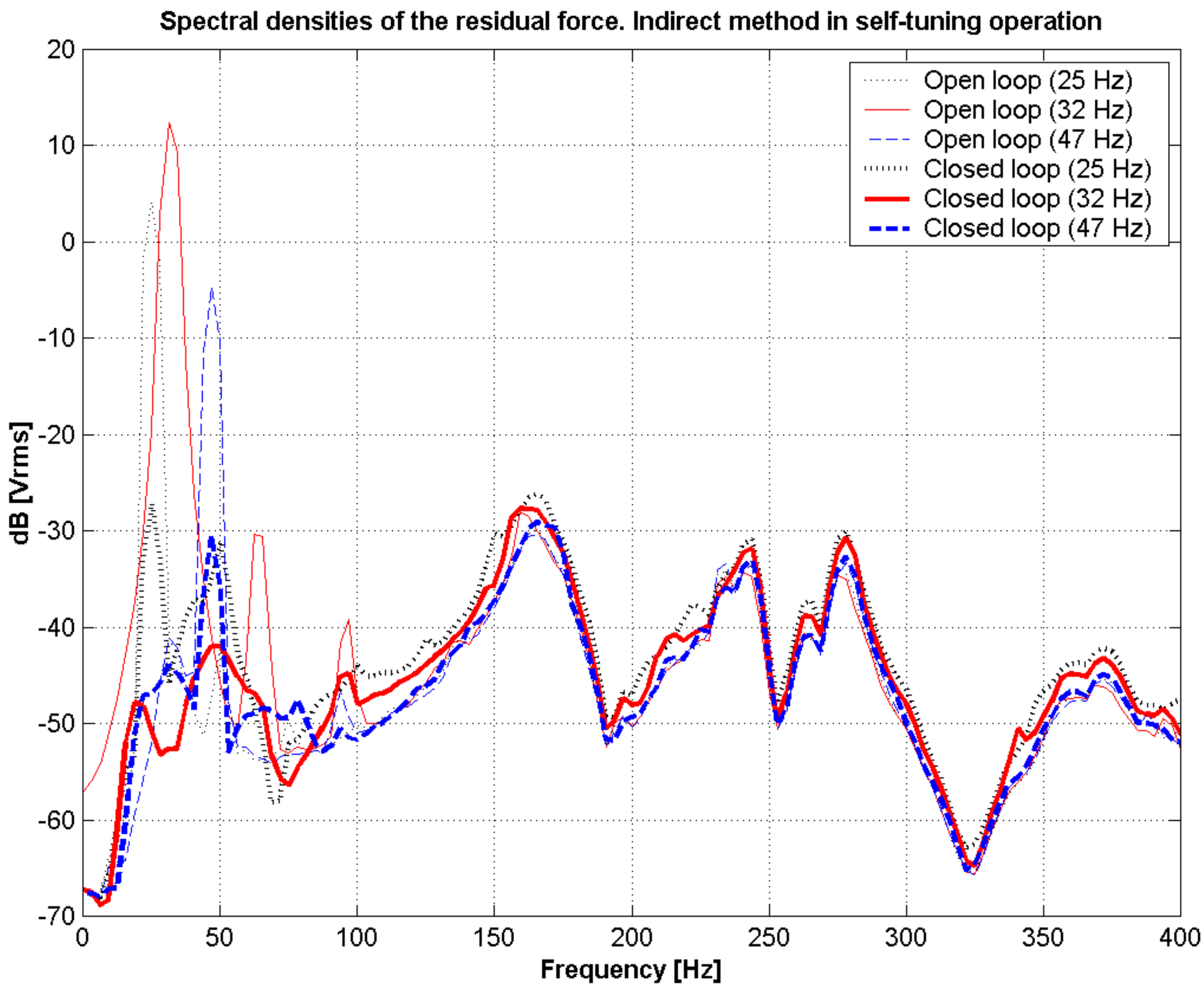
Implementation protocol 2: Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample

Frequency domain results – direct adaptive method

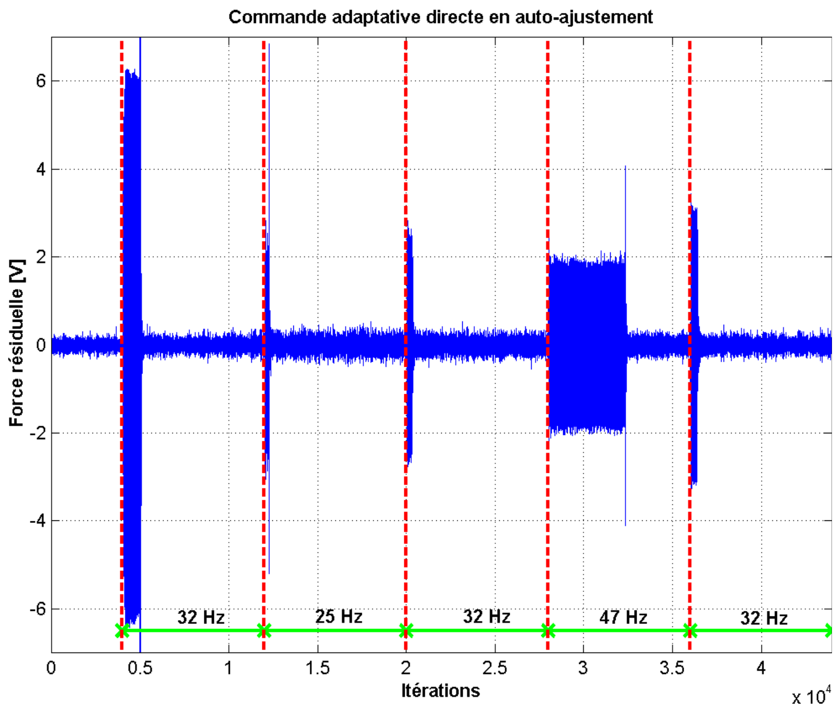


Frequency domain results – indirect adaptive method

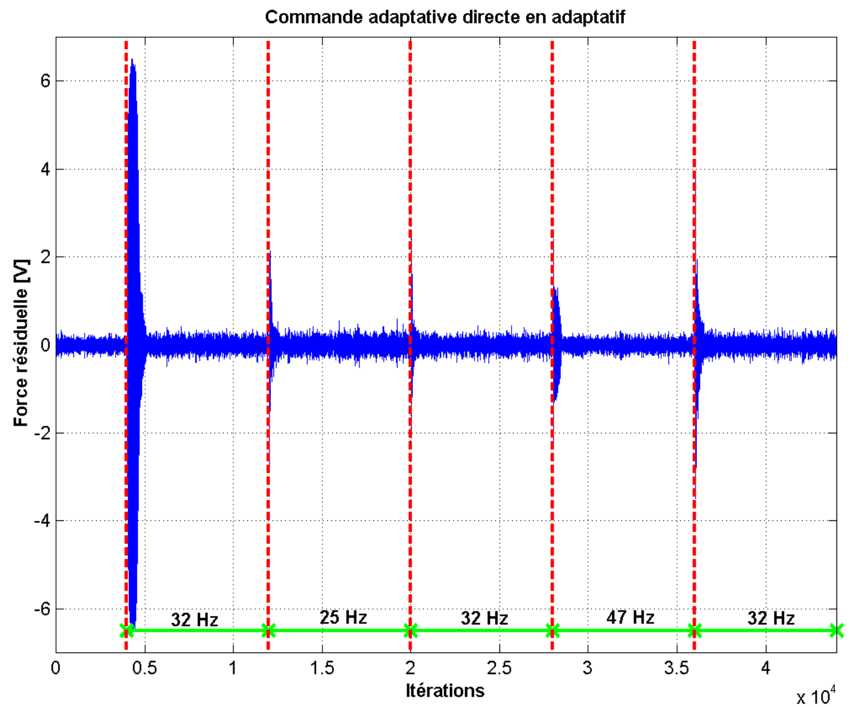


Direct Adaptive Control

Self-tuning Mode



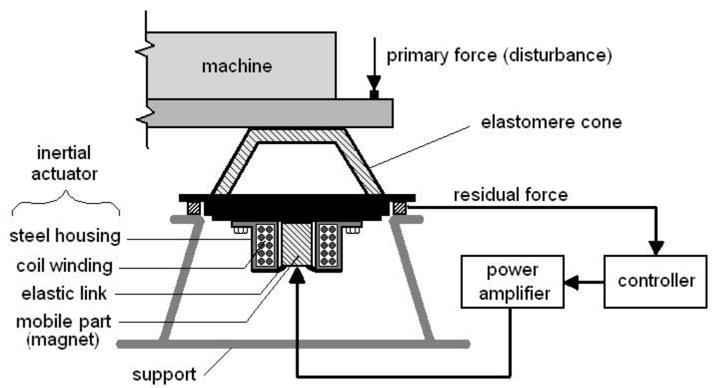
Adaptive Mode



- *Direct adaptive control in adaptive mode operation gives better results than direct adaptive control in self-tuning mode*
- *Direct adaptive control leads to a much simpler implementation and better performance than indirect adaptive control*

Active vibration control using an inertial actuator
Real-time results

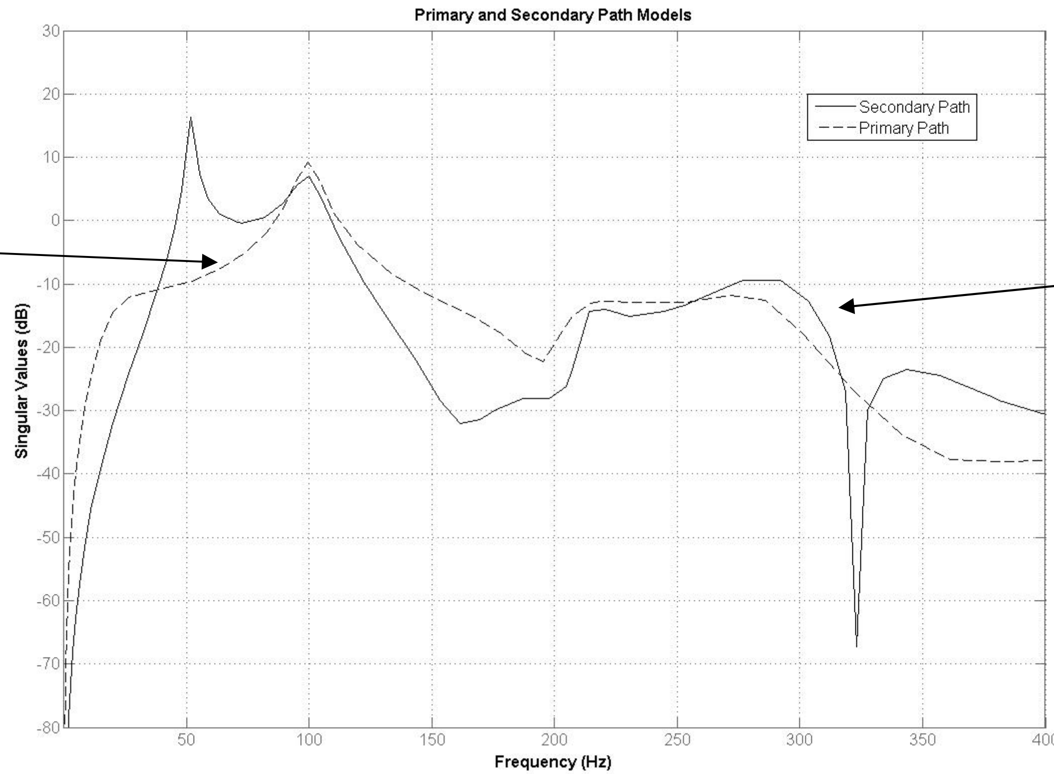
Rejection of two simultaneous sinusoidal disturbances



Active vibration control using an inertial actuator

Frequency Characteristics of the Identified Models

Primary path

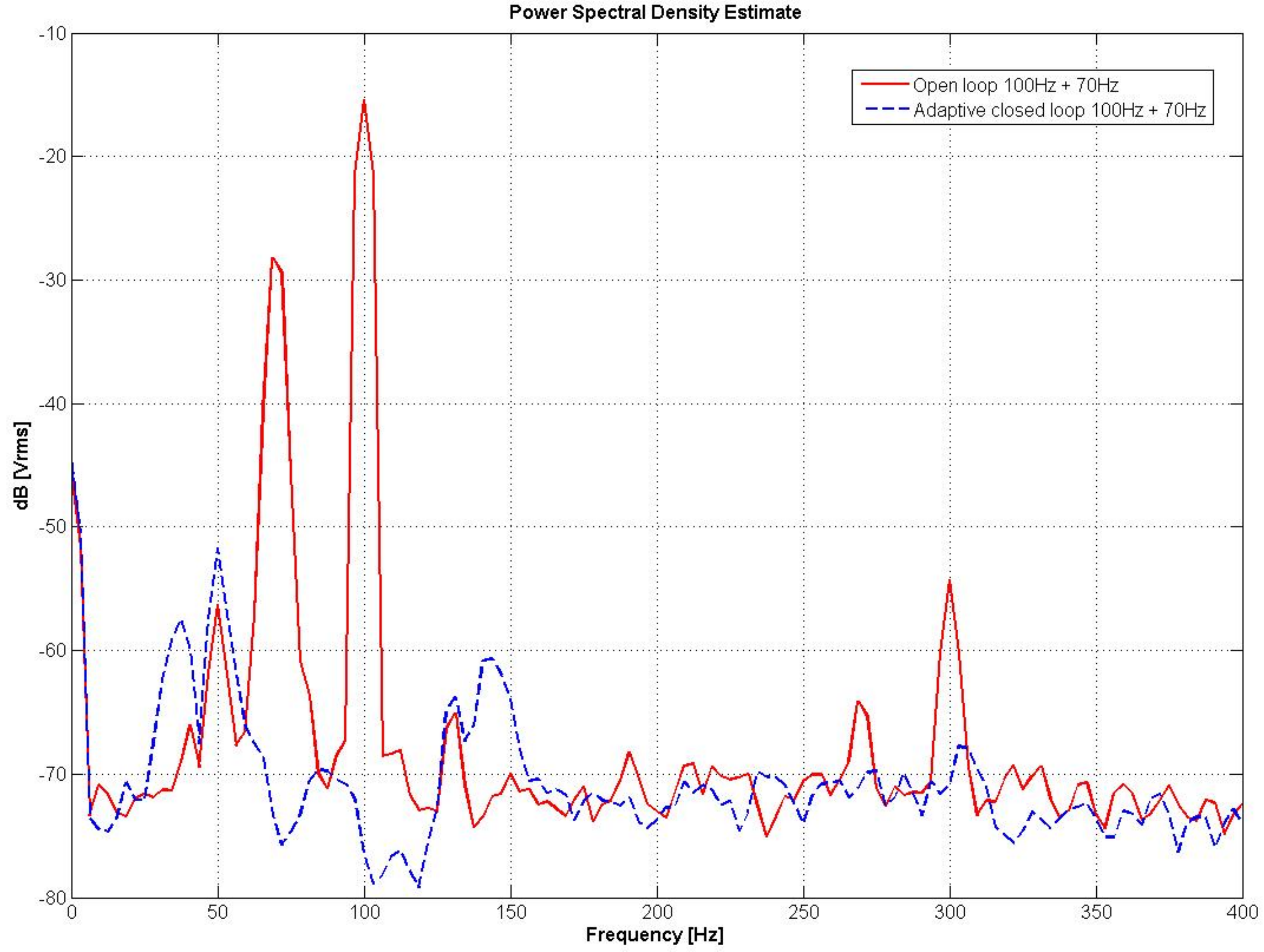


Secondary path

Complexity of secondary path: $n_A = 10 \,;\, n_B = 12 \,;\, d = 0$

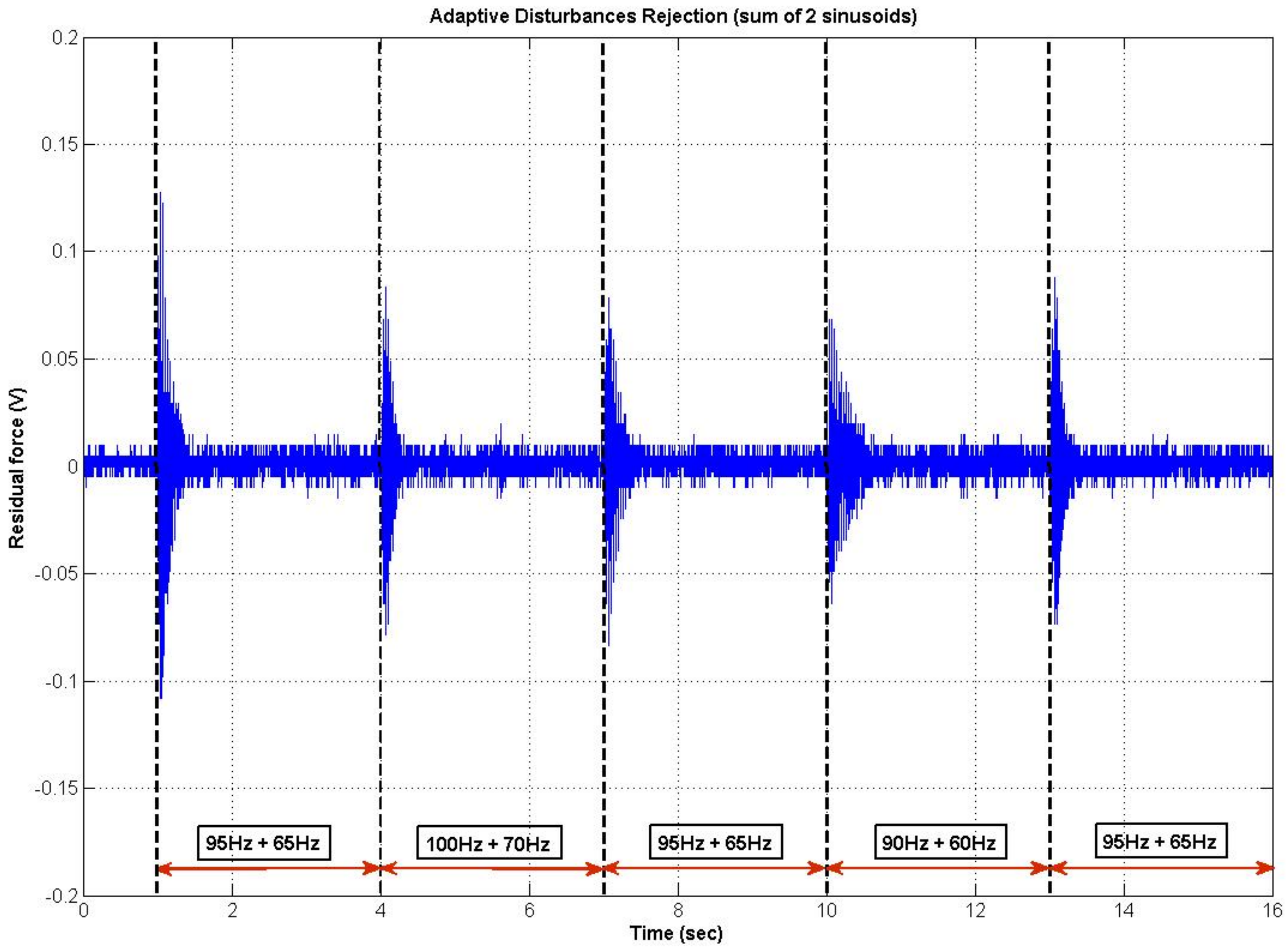
Frequency domain results – direct adaptive method

Rejection of two simultaneous sinusoidal disturbances



Time Domain Results – Direct adaptive control
Adaptive Operation

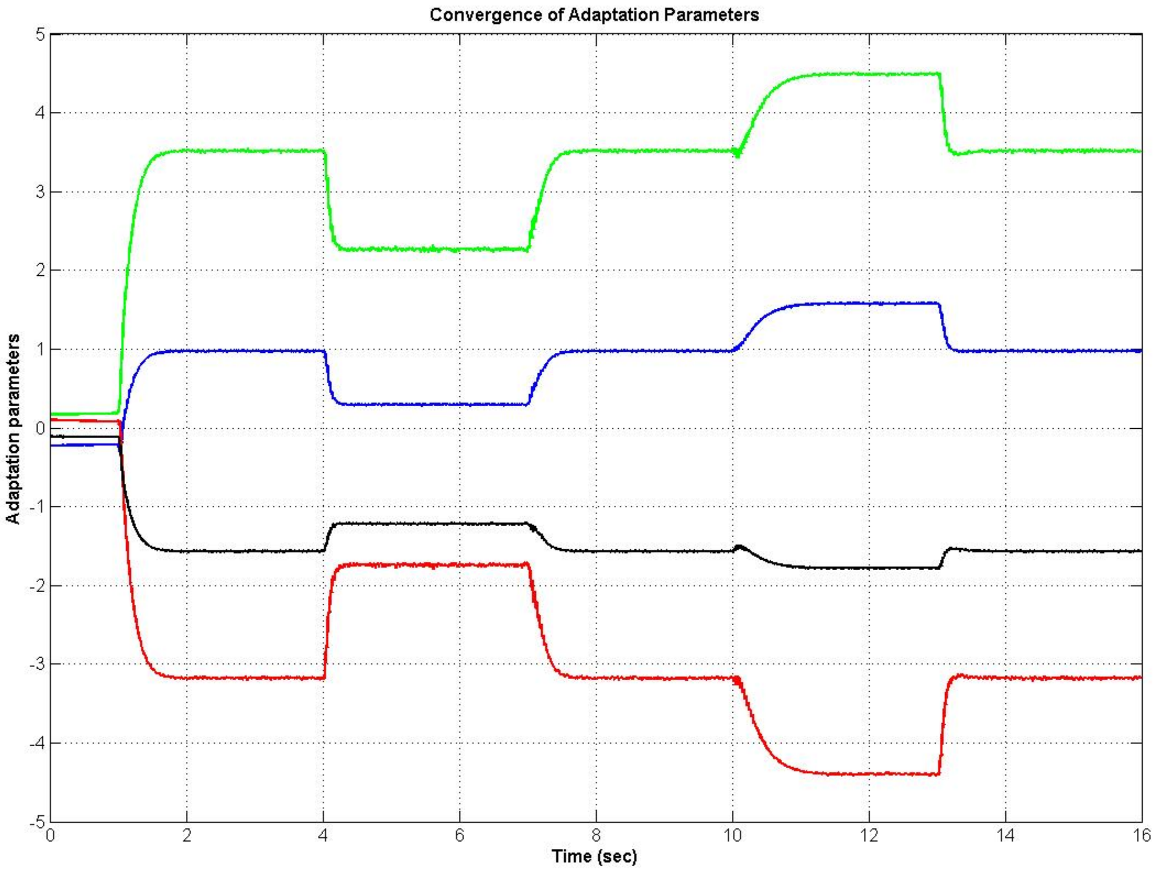
Simultaneous rejection of two time varying sinusoidal disturbances



Output

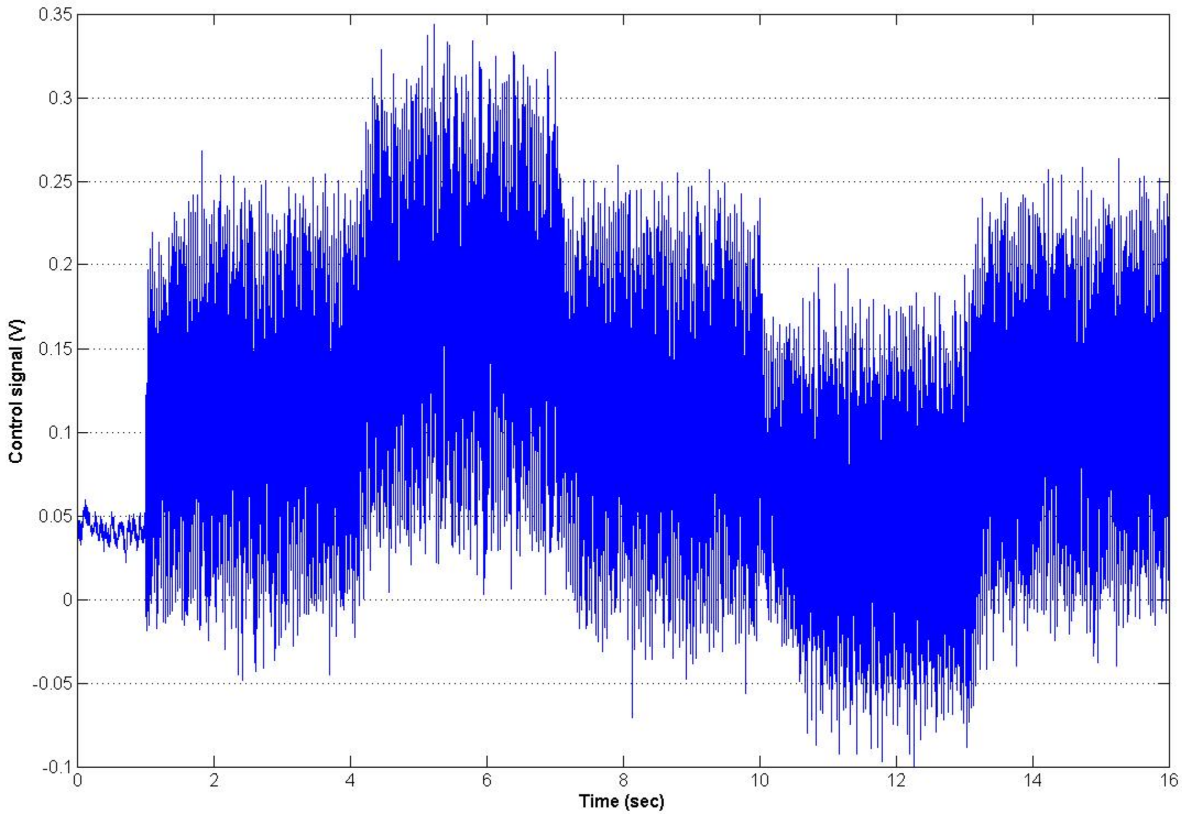
Time Domain Results – Direct adaptive control
Evolution of the Q parameters

Simultaneous rejection of two time varying sinusoidal disturbances



Time Domain Results – Direct adaptive control
Evolution of the control input

Simultaneous rejection of two time varying sinusoidal disturbances

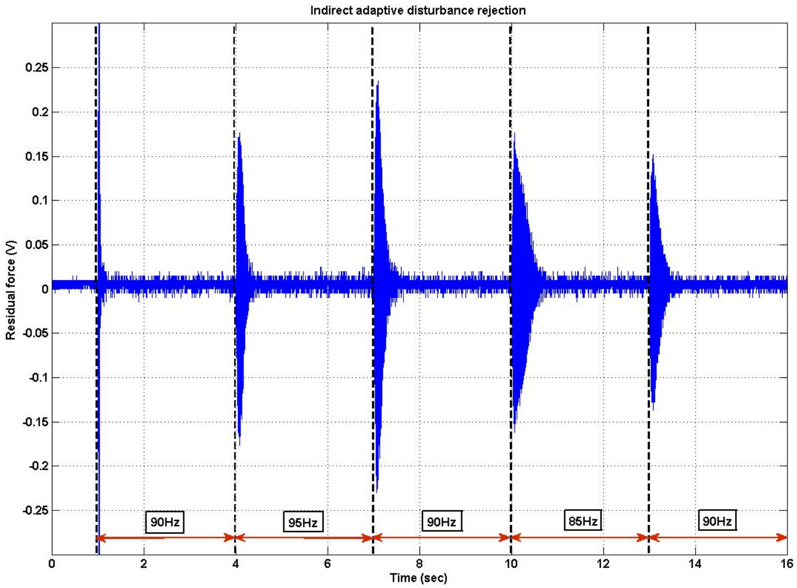


Comparison direct/indirect adaptive regulation

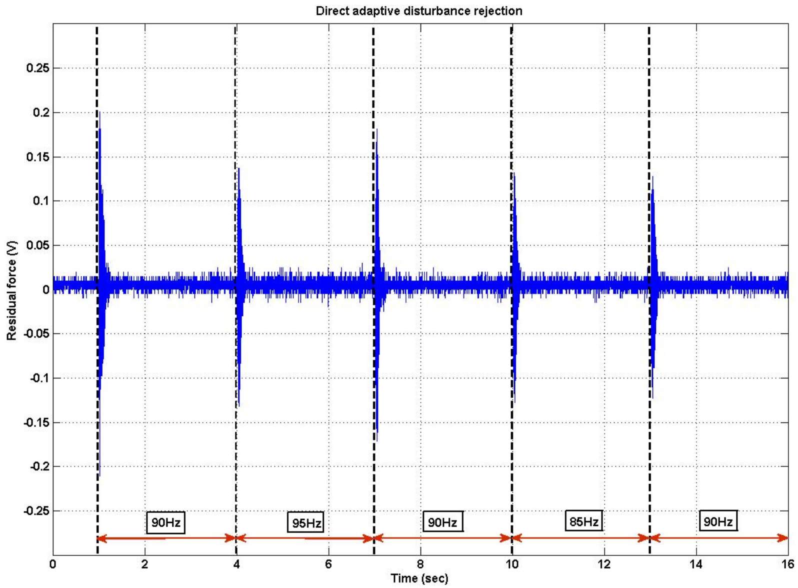
Time domain results – Adaptive regime

Active vibration control using an inertial actuator

Indirect adaptive method



Direct adaptive method



Direct *adaptive control leads to a much simpler implementation and better performance than* ***Indirect*** *adaptive control*

Conclusions

-Using internal model principle, adaptive feedback control solutions can be provided for the rejection of unknown disturbances

-Both direct and indirect solutions can be provided

-Two modes of operation can be used : self-tuning and adaptive

-Direct adaptive control is the simplest to implement

-Direct adaptive control offers better performance

-The methodology has been extensively tested on:
- active suspension system
- active vibration control with an inertial actuator